Chapter 2 Review Exercises

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1–14, find the limits.

1. $\lim_{x\to -2}(x^3 - 2x^2 + 1)$

2. $\lim_{x\to -2}\dfrac{x^2+1}{3x^2-2x+5}$

3. $\lim_{x\to 4}\sqrt{1-2x}$

4. $\lim_{x\to 5}\sqrt[3]{9-x^2}$

5. $\lim_{x\to 0}\dfrac{\dfrac{1}{2+x}-\dfrac{1}{2}}{x}$

6. $\lim_{x\to\pm\infty}\dfrac{2x^2+3}{5x^2+7}$

7. $\lim_{x\to\pm\infty}\dfrac{x^4+x^3}{12x^3+128}$

8. $\lim_{x\to 0}\dfrac{\sin 2x}{4x}$

9. $\lim_{x\to 0} x\csc x + 1$

10. $\lim_{x\to 0} e^x \sin x$

11. $\lim_{x\to 7/2^+} \text{int}\,(2x-1)$

12. $\lim_{x\to 7/2^-} \text{int}\,(2x-1)$

13. $\lim_{x\to\infty} e^{-x}\cos x$

14. $\lim_{x\to\infty}\dfrac{x+\sin x}{x+\cos x}$

In Exercises 15–20, determine whether the limit exists on the basis of the graph of $y = f(x)$. The domain of f is the set of real numbers.

15. $\lim_{x\to d} f(x)$

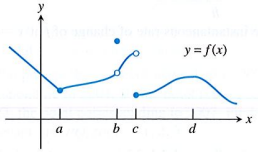
16. $\lim_{x\to c^+} f(x)$

17. $\lim_{x\to c^-} f(x)$

18. $\lim_{x\to c} f(x)$

19. $\lim_{x\to b} f(x)$

20. $\lim_{x\to a} f(x)$



In Exercises 21–24, determine whether the function f used in Exercises 15–20 is continuous at the indicated point.

21. $x = a$

22. $x = b$

23. $x = c$

24. $x = d$

In Exercises 25 and 26, use the graph of the function with domain $-1 \le x \le 3$.

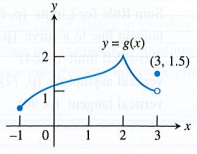
25. Determine

(a) $\lim_{x\to 3^-} g(x)$. (b) $g(3)$.

(c) whether $g(x)$ is continuous at $x = 3$.

(d) the points of discontinuity of $g(x)$.

(e) **Writing to Learn** whether any points of discontinuity are removable. If so, describe the new function. If not, explain why not.



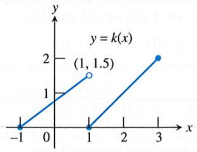
26. Determine

(a) $\lim_{x\to 1^-} k(x)$. (b) $\lim_{x\to 1^+} k(x)$. (c) $k(1)$.

(d) whether $k(x)$ is continuous at $x = 1$.

(e) the points of discontinuity of $k(x)$.

(f) **Writing to Learn** whether any points of discontinuity are removable. If so, describe the new function. If not, explain why not.



In Exercises 27 and 28, **(a)** find the vertical asymptotes of the graph of $y = f(x)$, and **(b)** describe the behavior of $f(x)$ to the left and right of any vertical asymptote.

27. $f(x) = \dfrac{x+3}{x+2}$

28. $f(x) = \dfrac{x-1}{x^2(x+2)}$

In Exercises 29 and 30, answer the questions for the piecewise-defined function.

29. $f(x) = \begin{cases} 1, & x \le -1 \\ -x, & -1 < x < 0 \\ 1, & x = 0 \\ -x, & 0 < x < 1 \\ 1, & x \ge 1 \end{cases}$

(a) Find the right-hand and left-hand limits of f at $x = -1$, 0, and 1.

(b) Does f have a limit as x approaches -1? 0? 1? If so, what is it? If not, why not?

(c) Is f continuous at $x = -1$? 0? 1? Explain.

30. $f(x) = \begin{cases} |x^3 - 4x|, & x < 1 \\ x^2 - 2x - 2, & x \ge 1 \end{cases}$

(a) Find the right-hand and left-hand limits of f at $x = 1$.

(b) Does f have a limit as $x \to 1$? If so, what is it? If not, why not?

(c) At what points is f continuous?

(d) At what points is f discontinuous?

In Exercises 31 and 32, find all points of discontinuity of the function.

31. $f(x) = \dfrac{x+1}{4-x^2}$

32. $g(x) = \sqrt[3]{3x+2}$

In Exercises 33–36, find **(a)** a power function end behavior model and **(b)** any horizontal asymptotes.

33. $f(x) = \dfrac{2x+1}{x^2-2x+1}$

34. $f(x) = \dfrac{2x^2+5x-1}{x^2+2x}$

35. $f(x) = \dfrac{x^3-4x^2+3x+3}{x-3}$

36. $f(x) = \dfrac{x^4-3x^2+x-1}{x^3-x+1}$

In Exercises 37 and 38, find **(a)** a right end behavior model and **(b)** a left end behavior model for the function.

37. $f(x) = x + e^x$

38. $f(x) = \ln|x| + \sin x$

Group Activity In Exercises 39 and 40, what value should be assigned to k to make f a continuous function?

39. $f(x) = \begin{cases} \dfrac{x^2+2x-15}{x-3}, & x \ne 3 \\ k, & x = 3 \end{cases}$

40. $f(x) = \begin{cases} \dfrac{\sin x}{2x}, & x \ne 0 \\ k, & x = 0 \end{cases}$

Group Activity In Exercises 41 and 42, sketch a graph of a function f that satisfies the given conditions.

41. $\lim_{x\to\infty} f(x) = 3$, $\lim_{x\to-\infty} f(x) = \infty$, $\lim_{x\to 3^+} f(x) = \infty$, $\lim_{x\to 3^-} f(x) = -\infty$

42. $\lim_{x\to 2} f(x)$ does not exist, $\lim_{x\to 2^+} f(x) = f(2) = 3$

43. *Average Rate of Change* Find the average rate of change of $f(x) = 1 + \sin x$ over the interval $[0, \pi/2]$.

44. *Rate of Change* Find the instantaneous rate of change of the volume $V = (1/3)\pi r^2 H$ of a cone with respect to the radius r at $r = a$ if the height H does not change.

45. *Rate of Change* Find the instantaneous rate of change of the surface area $S = 6x^2$ of a cube with respect to the edge length x at $x = a$.

46. *Slope of a Curve* Find the slope of the curve $y = x^2 - x - 2$ at $x = a$.

47. *Tangent and Normal* Let $f(x) = x^2 - 3x$ and $P = (1, f(1))$. Find **(a)** the slope of the curve $y = f(x)$ at P, **(b)** an equation of the tangent at P, and **(c)** an equation of the normal at P.

48. *Horizontal Tangents* At what points, if any, are the tangents to the graph of $f(x) = x^2 - 3x$ horizontal? (See Exercise 47.)

49. *Bear Population* The number of bears in a federal wildlife reserve is given by the population equation

$$p(t) = \frac{200}{1 + 7e^{-0.1t}},$$

where t is in years.

(a) **Writing to Learn** Find $p(0)$. Give a possible interpretation of this number.

(b) Find $\lim_{t\to\infty} p(t)$.

(c) **Writing to Learn** Give a possible interpretation of the result in part (b).

50. *Taxi Fares* Bluetop Cab charges \$3.20 for the first mile and \$1.35 for each additional mile or part of a mile.

(a) Write a formula that gives the charge for x miles with $0 \le x \le 20$.

(b) Graph the function in (a). At what values of x is it discontinuous?

51. Table 2.4 gives the population of Florida for several years. All data was collected on July 1 of the given year.

TABLE 2.4 Population of Florida

Year	Population (in thousands)
2000	16,047
2002	16,341
2004	17,314
2006	18,019
2008	18,328
2009	18,538

Source: U.S. Census Bureau, *Statistical Abstract of the United States; 2009–2010.*

(a) Let $x = 0$ represent 2000, $x = 1$ represent 2001, and so forth. Make a scatter plot for the data.

(b) Let P represent the point corresponding to 2009, Q_1 the point corresponding to 2002, Q_2 the point corresponding to 2004, and Q_3 the point corresponding to 2008. Find the slope of the secant the PQ_i for $i = 1$, 2, and 3.

(c) Using the same information given in part (b), find the average rates of change from Q_i to P.

(d) Estimate the instantaneous rate of change of the population on July 1, 2009.

(e) **Writing to Learn** Assuming the population growth in Table 2.4 *is linear*, estimate the population of Florida in 2020. Explain why linear growth may or may not be a bad assumption over longer periods of time.

52. *Limit Properties* Assume that

$$\lim_{x\to c}[f(x) + g(x)] = 2,$$
$$\lim_{x\to c}[f(x) - g(x)] = 1,$$

and that $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ exist. Find $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$.

AP* Examination Preparation

53. Free Response Let $f(x) = \dfrac{x}{|x^2-9|}$.

(a) Find the domain of f.

(b) Write an equation for each vertical asymptote of the graph of f.

(c) Write an equation for each horizontal asymptote of the graph of f.

(d) Is f odd, even, or neither? Justify your answer.

(e) Find all values of x for which f is discontinuous and classify each discontinuity as removable or nonremovable.

54. Free Response Let $f(x) = \begin{cases} x^2 - a^2x & \text{if } x < 2, \\ 4 - 2x^2 & \text{if } x \ge 2. \end{cases}$

(a) Find $\lim_{x\to 2^-} f(x)$.

(b) Find $\lim_{x\to 2^+} f(x)$.

(c) Find all values of a that make f continuous at 2. Justify your answer.

55. Free Response Let $f(x) = \dfrac{x^3-2x^2+1}{x^2+3}$.

(a) Find all zeros of f.

(b) Find a right end behavior model $g(x)$ for f.

(c) Determine $\lim_{x\to\infty} f(x)$ and $\lim_{x\to\infty} \dfrac{f(x)}{g(x)}$.